The Lee Fields Medal

Inaugural Competition, Maths Week 2018

Department of Mathematics, CIT

Time Allowed: Up to Three Hours

Tables and calculators may be used.

Answer all ten questions

1. Using each number exactly once, place the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in a 3×3 square so that the rows, columns and diagonals sum to the same total.

2. Professor Oldie does not believe in calculators. You have to prove it to him on paper, using mathematical considerations, that

$$\sqrt{10} > \sqrt{2} + \sqrt{3}.$$

   You may not use approximations nor your calculator.

3. CIT students Rebecca and Aoife share an apartment which is 6 km away from college. One day they left the apartment at the same time but decided to walk to the college separately. Rebecca walked first half of the *distance* with the speed of 4 km/h and the second half of the *distance* at 2 km/h. Aoife had a different plan. She walked the first half of the *time* with the speed of 4 km/h and walked the remaining half of the *time* at 2 km/h. Who got to college first? Show all calculations.

4. Explain geometrically why the set of simultaneous equations

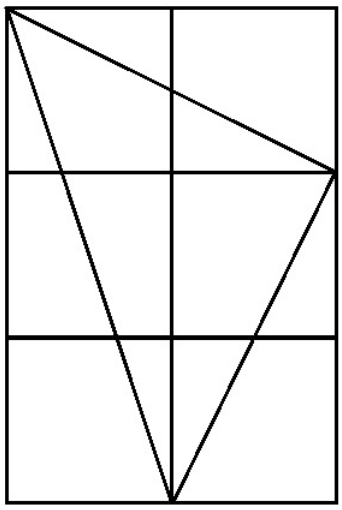
$$2x - y = -3$$
$$2x - y = -2$$

   has *no* solution.

5. Which is a closer fit, a square peg in a round hole, or a round peg in a square hole? Justify your answer fully.

6. Use the below figure to prove that

$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi = 180^\circ.$$



7. If you have 23 people in a room, what is the probability that at least two of them share a birthday?

8. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is maximised?

9. If every person in a group of 20 shook hands with all of the other people in the group, how many more handshakes take place than if the group were to split themselves into two groups of 10 and each person only shook hands with the other nine people in their group?

10. There are eight identical-looking coins; one of these coins is counterfeit and is known to be lighter than the genuine coins. What is the minimum number of weighings needed to identify the fake coin with a two-pan balance scale without weights?